

CERN-PH-TH/2004-159
hep-ph/0408269

Loop Calculations: Summary^{*}

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Abstract

Current and future colliders will provide high precision experimental data. In order to use the high experimental precision it has to be matched with theoretical predictions at the same level of accuracy or better. This involves the calculation of loop corrections at increasingly higher order. We briefly review the status of the field, especially in view of the calculations presented at this conference. We give an outlook about the theoretical requirements for the anticipated precision of a future e^+e^- linear collider.

^{*}plenary talk given at the “International Conference on Linear Colliders”, April 2004, Paris, France

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LOOP CORRECTIONS: SUMMARY

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Current and future colliders will provide high precision experimental data. In order to use the high experimental precision it has to be matched with theoretical predictions at the same level of accuracy or better. This involves the calculation of loop corrections at increasingly higher order. We briefly review the status of the field, especially in view of the calculations presented at this conference. We give an outlook about the theoretical requirements for the anticipated precision of a future e^+e^- linear collider.

1 Introduction

Past experiments have reached a precision that their results disagreed with the lowest-order Standard Model (SM) calculation. One example is the mass of the W boson, which can be predicted (at tree-level) in terms of the Z boson mass, the Fermi constant and the fine structure constant. However, only by taking the full 1-loop and leading 2- (and 3-) loop corrections into account (see e.g. Refs.^{1,2} for recent calculations), the experimentally measured value from LEP2³ is in agreement with the SM prediction. Another example is the cross section for $e^+e^- \rightarrow W^+W^-$. Only by the inclusion of the double-pole approximation 1-loop result⁴ the experimental result from LEP2³ and SM prediction are in good agreement. As a last example we take the anomalous magnetic moment of the muon. The final experimental result of the Brookhaven “Muon $g - 2$ Experiment” (E821)⁵ has a precision of 6×10^{-10} , the same size as (and thus being sensitive to) the SM electroweak (EW) 2-loop contributions⁶.

The currently operating Tevatron and other (low energy) experiments have the potential to improve current precisions. The upcoming LHC, besides being a discovery machine for new physics, will also improve precision measurements. Finally the prospective e^+e^- linear collider (LC) will provide measurements of masses, couplings and cross sections at (or even below) the per-cent level⁷. A special example is the GigaZ option, which will determine the W boson mass with an error of 7 MeV and the effective leptonic weak mixing angle with a precision of 1.3×10^{-5} , see Refs.^{8,9} and refs. therein. These anticipated future high precision measurements can only be utilized if they are matched with a theory prediction at the same level of accuracy or better. These theory predictions have to be obtained in the model under investigation (e.g. the SM, or the Minimal Supersymmetric SM (MSSM), for recent overviews see Refs.^{10,11}).

2 Status Of The Field

The status of the field of loop calculations is briefly summarized in Fig. 1. The complication of a higher-order loop calculation increases with the number of loops as well as with the number of external legs. On the other hand, it also increases with the number of (mass) scales appearing in the loop integral. While one-scale integrals (as occur e.g. in QCD) are usually the easiest possibility for a certain loop topology, two or more scales make the evaluation increasingly difficult. This poses a special problem in EW calculations, where many independent mass scales can appear in a single loop diagram. In Fig. 1 on the horizontal axis the number of legs, and on the vertical one the number of loops is shown. Accordingly, the number of scales has to be kept in mind for each individual case presented below.

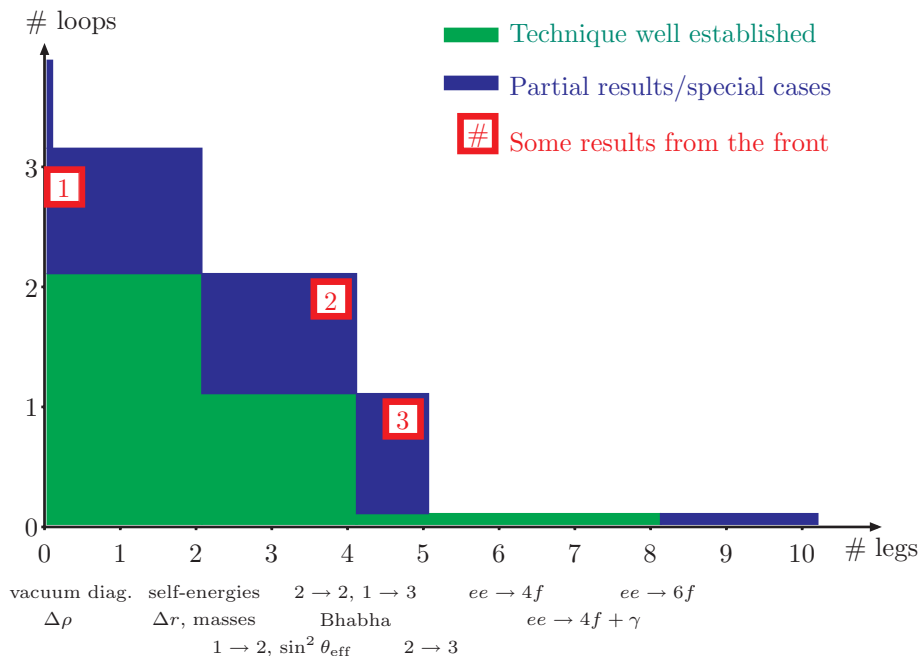


Figure 1: The current status of the field of loop calculations is shown. The medium shaded (green) area shows the well established techniques. The dark shaded (blue) area corresponds to the number of legs and loops where partial results or calculations for special cases are known. Also indicated with squares and numbers are some recently obtained results.

The medium shaded (green) area in Fig. 1 displays the number of loops and legs for which the techniques are meanwhile well established, even for an

arbitrary number of scales. For these cases often public algebraic computer codes exist that do the main part of the calculation itself, for an overview see Ref.¹⁰. The dark shaded (blue) area corresponds to the number of loops and legs for which partial results or calculations for special cases have been performed. This represents today's frontier of the field of loop calculations. For the sake of brevity we mention only three results from the front, indicated by the squares and numbers. # 1 is an example for a 3-loop calculation of vacuum diagrams (i.e. with no external legs) and one scale: the leading 3-loop EW corrections to the ρ parameter in the SM². Translated to precision observables, these corrections eliminated a theoretical uncertainty of $4 - 5$ MeV in the SM prediction of M_W and $2 - 3 \times 10^{-5}$ in the effective weak leptonic mixing angle, $\sin^2 \theta_{\text{eff}}$, see Sect. 1. # 2 indicates the progress made over the recent years in the evaluation of (massless) 2-loop box calculations obtained by several groups¹². These calculations are especially important e.g. for jet physics at the LHC and LC. # 3 represents the progress made in the last two years in the evaluation of the full 1-loop EW corrections (implying an arbitrary number of scales) to $2 \rightarrow 3$ processes. The full 1-loop calculation of $e^+e^- \rightarrow t\bar{t}H$ has been obtained by three independent groups¹³. These corrections are indispensable in order to match the anticipated precision of a future LC. Finally we would like to mention the progress made in the automated reduction of loop integrals to basic sets of integrals. The Integration By Parts (IBP) method¹⁴ allows to obtain a set of linear equations for different loop integrals. Following the method proposed in Ref.¹⁵, recently the first public code became available¹⁶ that (in principle) can perform the reduction to master integrals for arbitrary loop diagrams.

3 Contributions In Paris

In order to match the anticipated experimental precision of a future LC, the field of loop calculations has to advance substantially, as will be further discussed in the next section. The necessary improvement is indicated in Fig. 2 as the light shaded (orange) areas, which will have to be under full control for the LC precision. Some advance has been presented at this conference, which is shown as black rectangles (and numbers).

Three calculations of 1-loop corrections to $2 \rightarrow 2$ processes have been presented. A SM (re)calculation of deep inelastic neutrino scattering as observed in the NuTeV experiment¹⁷ has been performed¹⁸, using the automatized SANC system¹⁹. The comparison with another recent calculation²⁰ is still ongoing. For the same process the complete calculation of all SUSY 1-loop contributions has been performed²¹, finding that SUSY cannot explain the NuTeV

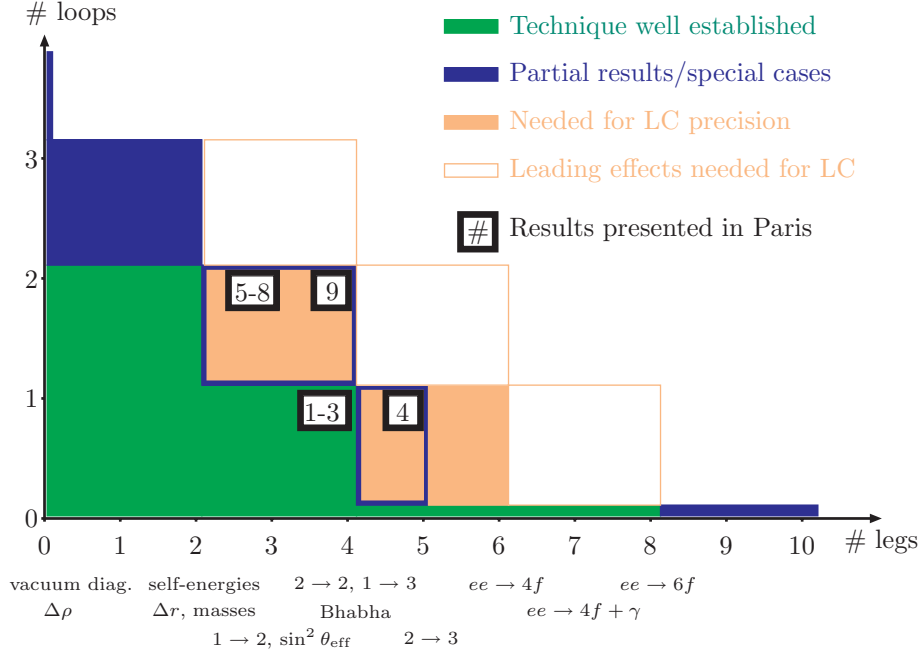


Figure 2: Besides the current status of the field and partial/special results (see also Fig. 1), the light shaded (orange) area shows what will be needed to match the anticipated LC precision. The squares and numbers indicate the contributions presented at this conference.

anomaly, see also Ref. ²². Moreover, a new recursive algorithm for the numerical evaluation of 1-loop tensor integrals has been presented ²³. This algorithm is supposed to be applicable to 1-loop diagrams with an arbitrary number of external legs, but so far has been applied to four.

A recalculation of the 1-loop QED corrections to $e^+e^- \rightarrow 2f + \gamma$ (i.e. with five external legs, however, only box and triangle diagrams contribute) in the SM has been presented ²⁴. This process contributes to Bhabha scattering, which as to be under control, since it serves as a luminosity monitor at the LC. The comparison with different other results showed agreement up to 10^{-5} .

Four new calculations with three external legs at the 2-loop level have been presented: First, the evaluation of EW logs of the type $\alpha^L \log^K(s/M^2)$ for $1 \rightarrow 2$ SM processes has been automatized ²⁶ (where s is the center-of-mass energy, M is the mass of a SM gauge boson, and $s \gg M^2$). Second, an example for the evaluation of the (subleading) Sudakov logs in EW processes has been performed ²⁷. The form factor for a $1 \rightarrow 2$ process within a massive $U(1)$ theory has been evaluated in the high-energy limit, being of the

form $\alpha^n \sum_{k=0}^4 \log^{2n-k}(s/M^2)$ for $n = 2$. Third, the evaluation of all 2-loop diagrams with a closed fermion loop within the SM for $\sin^2 \theta_{\text{eff}}$ has been presented²⁸. This calculation is especially relevant, since $\sin^2 \theta_{\text{eff}}$ is an important precision observable for the indirect determination of the Higgs boson mass^{3,9}. The new calculation eliminates a theoretical uncertainty of $\sim 4 \times 10^{-5}$ (see Sect. 1). Fourth, new SUSY 2-loop corrections to the anomalous magnetic moment of the muon have been shown²⁵. Since the SM shows a $2\text{--}3\sigma$ deviation from the experimental result, SUSY is a good candidate to accommodate this “discrepancy”. The 2-loop corrections are necessary to achieve a precision of the MSSM prediction of 1×10^{-10} , as has been done for the corresponding SM evaluation. It has been shown that the new evaluation can shift 2σ exclusion bounds in the MSSM parameter space substantially.

Finally the progress in the evaluation of 2-loop QED corrections for Bhabha scattering has been presented²⁹, being important for a future luminosity monitor at the LC, see above. This new calculation with one or two scales in each integral requires the evaluation of more than 40 new 2-loop box master integrals and has not been completed yet. This calculation is also an example of the application of the methods of Refs.^{14,15}.

4 Conclusions And Outlook

The requirement of loop calculations to match the LC precision is shown in Fig. 2 as the light shaded (orange) area. Light (orange) boxes indicate for which number of loops and legs at least leading effects will have to be available. One can see that the Paris contributions certainly advanced the field towards the required precision. However, they constitute the evaluation of special cases (sometimes possible due to a fortunate kinematical situation).

A lot of work remains to be done. Currently we are far away from e.g. a full EW evaluation of $2 \rightarrow 2$ processes at the 2-loop level (though considerable progress in the corresponding QED and QCD processes has been made). On the other hand, the high experimental precision achievable at a LC will be worthless if it cannot be matched with theoretical precisions at the same level of accuracy. Therefore the high-energy physics community has to continuously support the advance in this field to have the calculations at hand when the LC starts. Theoretical calculations should be viewed as an essential part of all current and future high-energy physics programs.

Acknowledgements

We thank S. Dittmaier for helpful discussions.

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